

The Power of Extrapolation in Federated Learning

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October 16, 2024



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Introduction

- ▶ Federated learning (FL) is a distributed training approach for machine learning models, where multiple clients collaborate under the guidance of a central server to optimize a loss function [1, 3].
- ▶ In this paper, We consider the following federated optimization problem,

$$\min_{x \in \mathbb{R}^d} \left\{ f(x) := \frac{1}{n} \sum_{i=1}^n f_i(x) \right\}, \quad (1)$$

where each $f_i : \mathbb{R}^d \mapsto \mathbb{R}$ is a differentiable function, n is the number of clients.

Introduction

- ▶ The most commonly used algorithm to address this problem is the federated average (FedAvg) [2, 3] algorithm. However, it suffers from client drift when the data is heterogeneous.
- ▶ In an attempt to tackle with this, FedProx was introduced li2020federated, which can be formulated as

$$x_{k+1} = \frac{1}{n} \sum_{i=1}^n \text{prox}_{\gamma f_i}(x_k). \quad (\text{FedProx})$$

- ▶ Compared with gradient based algorithms, proximal algorithms are more stable.

Introduction

- ▶ Proximal operators of a convex function can be viewed as projection to a certain level set of the function.
- ▶ It is known that the parallel projection methods for solving the convex feasibility problem is accelerated by a practice called extrapolation.

$$x_{k+1} = x_k + \alpha_k \left(\frac{1}{n} \sum_{i=1}^n \Pi_{\mathcal{X}_i}(x_k) - x_k \right). \quad (2)$$

Here $\alpha_k > 1$ is the extrapolation parameter, the intersection of convex sets $\mathcal{X}_i$ is assumed to be non-empty.

- ▶ This means that we move further along the line connecting the current iterate x_k and the average projection point $\frac{1}{n} \sum_{i=1}^n \Pi_{\mathcal{X}_i}(x_k)$.

Motivation

- ▶ In this paper, we assume that the proximal operators are solved exactly with no inaccuracies.
- ▶ Given the similarity between the proximal operator and the projection operator, we propose to use extrapolation with FedProx.

Assumptions

- ▶ (Interpolation) There exists $x_\star \in \mathbb{R}^d$ such that $\nabla f_i(x_\star) = 0$ for all $i \in [n]$.
- ▶ (Individual convexity) The function $f_i : \mathbb{R}^d \mapsto \mathbb{R}$ satisfies $0 \leq f_i(x) - f_i(y) - \langle \nabla f_i(y), x - y \rangle$, for all $x, y \in \mathbb{R}^d$.
- ▶ (Smoothness) The function $f_i : \mathbb{R}^d \mapsto \mathbb{R}$ satisfies $f_i(x) - f_i(y) - \langle \nabla f_i(y), x - y \rangle \leq \frac{L_i}{2} \|x - y\|^2$, for all $x, y \in \mathbb{R}^d$.

The interpolation assumption comes from the non-emptiness of convex feasibility.

Constant extrapolation

- For $\text{prox}_{\gamma f_i}(x_k)$, we know that the following identity holds,

$$\nabla M_{f_i}^{\gamma}(x_k) = \frac{1}{\gamma} (x_k - \text{prox}_{\gamma f_i}(x_k)),$$

where $M_{f_i}^{\gamma}(x)$ is the Moreau envelope of f_i . This allows us to formulate the algorithm as

$$x_{k+1} = x_k - \alpha_k \gamma \cdot \frac{1}{n} \sum_{i=1}^n \nabla M_{f_i}^{\gamma}(x_k),$$

which is running SGD towards $M^{\gamma}(x) := \frac{1}{n} \sum_{i=1}^n M_{f_i}^{\gamma}(x)$.

- The interpolation assumption guarantees that minimizers of f and M^{γ} coincide.

Constant extrapolation

Assume Assumption 1, 2 and 3 holds, a fixed $\alpha_k = \alpha \in (0, 2/\gamma L_{\gamma, \tau})$, minibatch of size τ , local stepsize γ , we have

$$\mathbb{E}[f(x_K)] - \inf f \leq C(\gamma, \tau, \alpha) \cdot \frac{\|x_0 - x_\star\|^2}{K}, \quad (3)$$

where

$$C(\gamma, \tau, \alpha) := \frac{1 + \gamma L_{\max}}{\alpha \gamma (2 - \alpha \gamma L_{\gamma, \tau})}$$
$$L_{\gamma, \tau} := \frac{n - \tau}{\tau(n - 1)} \frac{L_{\max}}{1 + \gamma L_{\max}} + \frac{n(\tau - 1)}{\tau(n - 1)} L_\gamma.$$

Here L_γ is the smoothness constant of $M^\gamma(x)$.

Remarks

- ▶ The optimal constant extrapolation parameter is $1/\gamma L_{\gamma,\tau} > 1$, resulting in $C(\gamma, \tau, \alpha_{\gamma,\tau}) = L_{\gamma,\tau} (1 + \gamma L_{\max}) \leq L_{\max}$, which indicates convergence.
- ▶ If we assume in addition that f is μ -strongly convex, we obtain linear convergence.
- ▶ The convergence rate of FedProx is given by $C(\gamma, \tau, 1)$, and we have

$$\frac{C(\gamma, \tau, 1)}{C(\gamma, \tau, \alpha_{\gamma,\tau})} \geq 2 + \gamma L_{\max} + \frac{1}{\gamma L_{\max}},$$

indicating the superiority of our algorithm compared to FedProx.

Adaptive extrapolation

Since the extrapolation parameter α_k is naturally connected to stepsize of SGD, we can use adaptive rules to determine it.

$$\alpha_{k,G} := \frac{\frac{1}{n} \sum_{i=1}^n \|x_k - \text{prox}_{\gamma f_i}(x_k)\|^2}{\left\| \frac{1}{n} \sum_{i=1}^n (x_k - \text{prox}_{\gamma f_i}(x_k)) \right\|^2} \geq 1. \quad (\text{GraDS})$$

$$\alpha_{k,S} := \frac{\frac{1}{n} \sum_{i=1}^n \left(M_{f_i}^{\gamma}(x_k) - \inf M_{f_i}^{\gamma} \right)}{\gamma \left\| \frac{1}{n} \sum_{i=1}^n \nabla M_{f_i}^{\gamma}(x_k) \right\|^2} \geq \frac{1}{2\gamma L_{\gamma}}. \quad (\text{StoPS})$$

Adaptive extrapolation

Assume assumption 1, 2 and 3 holds, if we are using $\alpha_k = \alpha_{k,G}$ in the full batch case, we have

$$\mathbb{E}[f(\bar{x}_K)] - \inf f \leq \frac{1 + \gamma L_{\max}}{2 + \gamma L_{\max}} \cdot \left(\frac{1}{\gamma} + L_{\max} \right) \cdot \frac{\|x_0 - x_\star\|^2}{\sum_{k=0}^{K-1} \alpha_{k,G}}, \quad (4)$$

where $\bar{x}_K$ is chosen randomly from the first K iterates $\{x_0, x_1, \dots, x_{K-1}\}$ with probabilities $p_k = \alpha_{k,G} / \sum_{k=0}^{K-1} \alpha_{k,G}$. Similarly, if we are using $\alpha_k = \alpha_{k,S}$, we have

$$\mathbb{E}[f(\bar{x}_K)] - \inf f \leq \left(\frac{1}{\gamma} + L_{\max} \right) \cdot \frac{\|x_0 - x_\star\|^2}{\sum_{k=0}^{K-1} \alpha_{k,S}}, \quad (5)$$

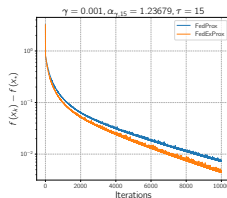
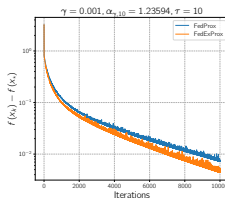
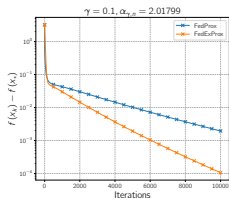
where $\bar{x}_K$ is chosen randomly from the first K iterates $\{x_0, x_1, \dots, x_{K-1}\}$ with probabilities $p_k = \alpha_{k,S} / \sum_{k=0}^{K-1} \alpha_{k,S}$.

Remarks

- ▶ We can extend the theorem into stochastic setting, using a stochastic version of the two adaptive stepsizes.
- ▶ Both FedExProx-GraDS and FedExProx-StoPS exhibits “semi-adaptivity”. A small γ hinders convergence, however, setting it to $\frac{1}{L_{\max}}$ limits the worsening of the convergence to a factor of 2.

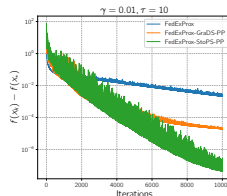
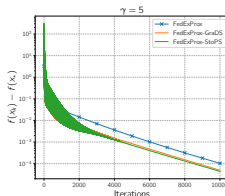
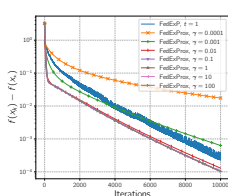
Experiments

- Comparison of FedProx and FedExProx in the full batch or minibatch setting.



Experiments

- Comparison of FedExP, FedExProx, FedExProx-GraDS and FedExProx-StoPS in terms of iteration complexity in the full batch or minibatch setting.



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